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NASA TECHNICAL TRANSLATION NASA TT F-15,373

HEAT TRANSFER BETWEEN LAMINAR LIQUID JET AND A FLAT SURFACE

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TRANSLATION OF

Israel Institute of Technology, M.Sc. Thesis, Technion, Haifa, 1964, 45 pages.

(NASA-TT-F-15373) HEAT TRANSFER BETWEEN LAMINAR LIQUID JET AND A FLAT SURFACE (Scientific Translation Service) ~~47~~ 53 p HC $5.50 CSCL 20D

N74-17971

Unclas

G3/12 31680

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 MARCH 1974

ANOTATION

The research was carried out under the supervision of Dr. A. Stotter, at the faculty of Mechanical Engineering.

The author wishes to thank Dr. A. Stotter who initiated and guided this research for his contribution to its success, and Professor J. F. Ury and Professor R. Peck who gave their advice in the beginning of the work, as well as to Dr. M. Bentwich and Dr. Chieger for their help at the later stages of the project. Further, I wish to thank for the help which I received in the faculty of Mechanical Engineering and in the Heat and Power Laboratory by the members of the academic staff headed by Professor R. Landsberg and by the technical staff headed by Mr. Z. Chechanover.

SYNOPSIS

The thesis describes the results of research on heat transfer and flow phenomena in a liquid jet, impinging on a flat perpendicular surface. In this investigation, the various zones of flow inside the impinging jet were examined, and the general character of the flow in each of them was established. Neglecting gravity and surface tension and using the order of magnitude analysis it was possible to specify the momentum equation for the viscous flow zone, where the thickness of the flowing layer is small compared with the distance from the impingement point, and where the velocity is mainly radial. These equations were solved by the assumption of the existence of a "similarity solution" which enabled the statement of two ordinary differential equations and their solutions. These solutions describe the stream function and the local layer thickness, and they enable the determination of the local velocity components.

Using assumptions similar to those described above, and substituting the calculated velocity, it was possible to state the appropriate energy equation. This equation was solved by separation of variables, the local temperature being a function of the surface temperature and distance from the surface. The heat transfer coefficient, Nusselt Number, and the mixing cup temperature were derived from the above solution.

Apart from the analytical study, an experimental investigation was initiated with the following scope:

1. qualitative confirmation of the basic assumptions;

2. qualitiative study of the flow;

3. quantitative confirmation of the theoretical solution.

By the simultaneous performance of the analytic and experimental studies, checking and improvement of the assumptions were possible. Comparison of calculated and measured values of heat transfer showed slight deviations only. A method to calculate the separation diameter of the jet from the flat surface was derived as an additional result.

As a result of this research, it is possible to calculate heat transfer between laminar impinging jet and a flat surface based on the following parameters: flow conditions in the free jet, properties of the flowing liquid, and the solid surface temperature.

SYMBOLS AND ABBREVIATIONS

a	constant (29)
a	constant (14a)
a_0	constant (52)
a_1	constant (53)
A	constant (49)
A_2	constant (29)
C_p	specific heat
d	2r
d_H	hydraulic diameter = $\left(\frac{\text{cross section of flow}}{\text{wet circumference}}\right) \times 4$
D	constant (18)
f	dimensionless stream function (26)
g	dimensionless thickness of flowing layer (25)
h	heat transfer coefficient
k	thermal conductivity
K	function (51)
L	function (51)
M	function (51)
Nu	Nusselt number
P	pressure
Pr	Prandtl number
q	dimensionless distance from impingement point (14)
q_t	heat flux
Q	volumetric flow rate of jet
Q	surface temperature (46) (only in Chapter 3)
r	distance from impingement point
Re	Reynolds number
t	temperature
u	velocity in the direction of r
v	velocity in the direction of y

V_i average velocity of the free jet

y distance from flat surface

Y temperature function (50)

Greek Letters

α thermal diffusion

λ constant (40, 47)

δ thickness of flowing layer

ζ dimensionless distance from flat surface in stagnant flow (2a)

η dimensionless distance from flat surface (11)

θ dimensionless temperature (36)

ν kinematic viscosity

ρ density

ϕ stream function in stagnant flow (1a)

ψ stream function

Indices and Notations

$(\ )_o$ values in the diameter d_o (in Chapter 1b)

$(\ )_{av}$ average value in the direction of y

$(\ )_i$ values in the free jet

$(\ )_{mix}$ average values in the liquid after separation

$(\ )_s$ values in the dameter of separation

$(\bar{\ })$ value on the flat surface

HEAT TRANSFER BETWEEN LAMINAR LIQUID JET AND A FLAT SURFACE

Micha Wolfshtein

CHAPTER A
FOREWORD

1. General Background

Gaseous or liquid jets find many uses in modern technology, /4* in turbines and jet engines. Indeed, many papers exist which deal with the conditions of flow in various jets (see [1, 2, 3]). It is worthwhile mentioning that the pressure exerted by a liquid jet impinging on a flat surface was measured by Reich [4]. He found also that the velocity after the impact with the flat surface is almost parallel to the surface.

The work of Glauert [5] merits special attention. He calculated analytically the distribution of the velocity in incompressible, steady, turbulent and laminar, two-dimensional gas jets having axial symmetry, which impinge on a flat surface. His work was checked experimentally by Bakke [6] and Bradshaw and Love [7], who proved that the description given by him is accurate a sufficient distance from the impingement point of the turbulent jet. The problem of the distribution of the velocity and pressure in the impact region is mentioned in the last reference, and is dealt with in the case of the turbulent gaseous jet.

Because of insufficient knowledge concerning the flow, we do not have enough data on heat transfer. Included in [8, 9, 10, 11, 12, 13, and 14] are results of various experiments which measure heat transfer between turbulent mixing jets and plane

* Translator's note: Numbers in margin indicate pagination in original foreign text.

surfaces. Wolfshtein and Stotter [15] tried to solve the heat transfer equation analytically, using the equation for the velocity which was developed by Glauert [5] for the case of an incompressible gas jet which mixes with its surrounding. The mentioned material does not provide sufficient information concerning the flow conditions and heat transfer in the impinging jets, and it is impossible to base an engineering design or an accurate scientific calculation on the information now available. This situation is in contrast to the many design problems which deal especially with heat transfer between impinging jets and plane surfaces. Among these problems are the problems of the turbulent jets of hot burning gases which are produced by missiles and impinge on the launching pads, turbulent jets of air for cooling industrial plants, laminar jets of oil for cooling engine pistons, and laminar jets for cooling or heating liquids in chemical plants.

2. Types of Jets

As a basis for an accurate analysis of flow and heat transfer in incompressible impinging jets, we suggest the following classification according to Wolfshtein and Stotter [15]:

1) a turbulent jet of a liquid having a viscosity similar to that of the surroundings, such as a gas jet in the atmosphere or a water jet in water.

2) laminar jet with conditions similar to paragraph 1. /5

3) turbulent jet of a liquid having a viscosity different by an order of magnitude from that of its surroundings, such as a water jet in the atmosphere.

4) laminar jet under conditions similar to paragraph 3.

Most of the mentioned references deal with jets which are included in paragraph 1. Glauert [5], Wolfshtein and Stotter [15] deal with paragraph 2. Research on cases mentioned in paragraphs 3 and 4 has not reached the author's attention, except for the research mentioned in [4], which deals especially with the distribution of pressures at the impingement point.

Considering the character of the problem, the following division is desired between two zones in any jet.

1) The impingement zone, in which the two components of the velocity have the same order of magnitude, and where changes in the direction of the momentum occur.

2) The parallel flow zone, in which the main component of the velocity is parallel to the surface, but the second component is much smaller. This zone is characterized by a large ratio of the distance to the impingement point and the distance to the solid body.

The mathematical methods which one uses in dealing with analytical solutions of flow differ considerably in the two mentioned zones because of the different characteristics of the simplifying assumptions which are used in the two zones. One may expect to have a difference between the two solutions and it will be necessary to make adjustments.

3. Definition of the Problem

The purpose of the present research is to suggest a method by which one may calculate heat transfer between a steady impinging laminar jet, which does not mix with its surrounding (having a viscosity much greater than its surrounding) and a plane surface perpendicular to it. The derived solutions are not suitable for the impingement point and its immediate surroundings but only to a zone which is sufficiently remote from the impingement point. The research is divided into four main parts.

1) analytic examination of the flow;

2) analytic examination of heat transfer;

3) experimental confirmation of the flow solution;

4) experimental confirmation of the heat transfer solution.

An intermediate result which was obtained in the course of this research but which will not be described here is the solution of the problem of heat transfer between a steady impinging jet which mixes with a surrounding having similar characteristics, and a plane surface which is perpendicular to it. A paper which describes the solution was published with the authorization of the Graduate School [15].

The following research will not deal with the influence of surface tension or the phenomena under discussion.

CHAPTER B
ANALYTIC SOLUTION OF THE FLOW PROBLEM /6

1. Assumptions and Methods of Solution

In order to solve the Navier-Stokes equation for the case of the impact of a steady laminar jet with a plane surface, it is necessary to make the following assumptions:

a) the influence of surface tension and gravity on the flow in the discussed zone is negligible;

b) the thickness of the flowing layer is much smaller than the distance to the impingement point;

c) the characteristics of the liquid (viscosity, density, heat conductivity, etc.) do not change considerably.

From the physical point of view, it is possible to divide the jet under consideration into four main parts as in Figure 1:

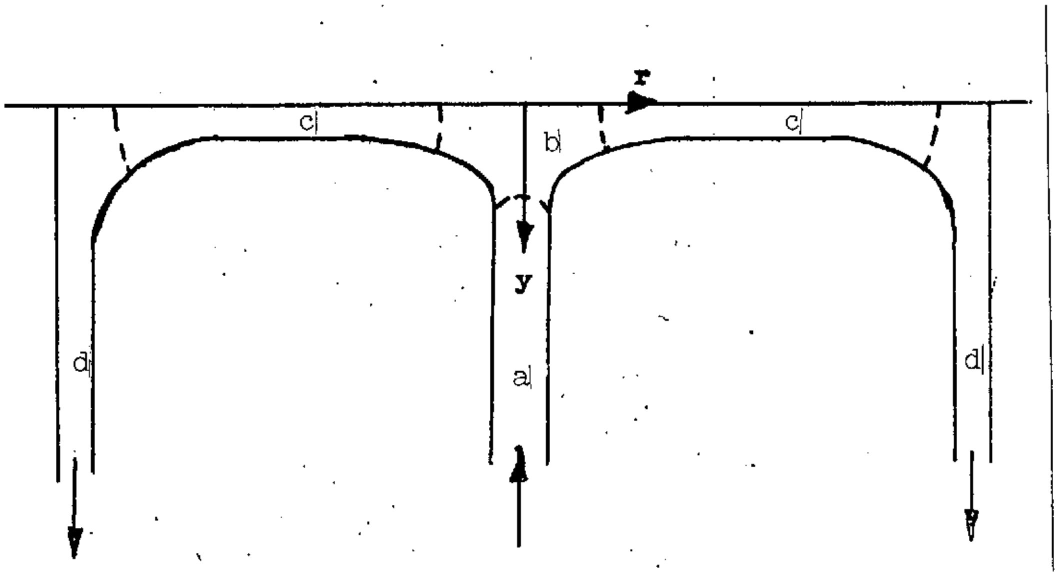

Figure 1. Impinging jet and division into zones.

a- free jet which moves with a constant velocity perpendicular to the plane surface; b- potential flow with a free surface on one side and a boundary layer on the plane surface from the other side in the zone of impact where the changes in direction of momentum and velocity occur; c- viscous flow with a free surface on one side and a plane solid surface on the other side and the velocity approximately parallel to the plane surface. In this region, the thickness of the flowing layer is small and therefore, the influence of the viscous forces is very great, which results in a considerable decrease in the velocity; d- viscous flow with a very low velocity.

As mentioned in the Foreword, the present research deals only with zone c. Assumption a is inappropriate for zone d, and assumption b is inappropriate for zones a and b. /7

An order of magnitude analysis of the Navier-Stokes equations in zone c, assuming a steady axially-symmetric flow, and by neglecting body forces (for a detailed description of this method of analysis, the reader is referred to the book of Schlichting [16], pp. 107 - 109), which yields the following results:

$$u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\frac{\partial P}{\partial y} = 0 \tag{2}$$

$$\frac{\partial}{\partial r}(u r) + \frac{\partial}{\partial y}(v r) = 0 \tag{3}$$

Since, on top of the free surface (where the liquid is in touch with the atmosphere), the pressure is constant, we get

$$\frac{\partial P}{\partial r} = 0$$

and therefore, in the present zone, we get

$$P = \text{Constant} \tag{4}$$

At this stage, it is worthwhile to mention that even though Equation (1) is typical to boundary layer phenomena, the flow in question is not a flow of a boundary layer in the usual sense, because the potential flow which usually exists at a sufficient distance from the boundary is totally missing here. As a result, there is a drastic reduction in the velocity, and there is a physical limit to the thickness of the layer, which depends on the existance of the continuity equation.

The initial conditions for the differential equations are:

For the diameter d_o, the thickness of the flowing layer is δ_o and the average radial velocity is Uo_{av}.

One can chose d_o arbitrarily with the following condition

$$\frac{\delta_o}{d_o} \ll 1 \tag{5}$$

Also, for the continuity equation to apply, we have /8

$$Q = \pi \delta_o d_o Uo_{av} \tag{6}$$

2. Mathematical Statement of the Flow Problem

The continuity equation (3) can immediately be integrated by defining a stream function ψ, and so

$$u = \frac{1}{r} \frac{\partial \psi}{\partial y} \tag{7}$$

$$v = -\frac{1}{r} \frac{\partial \psi}{\partial r} \tag{8}$$

The stream function has to fulfill the condition

$$\psi(\delta) - \psi(0) = 0$$

where δ is the thickness of the flowing layer which depends only on the radius. On the other hand, if a similarity solution exists, ψ has to be a unique function of the new variable η, and it is logical to define this function as follows:

$$\psi(\eta) - \psi(0) = \frac{Q}{2\pi} f(\eta) \tag{9}$$

where

$$f(1) = 1 \tag{10}$$

From this definition yields another condition

$$f(0) = 0 \tag{11}$$

Let us now define the layer thickness by

$$\delta = \delta_0 \, g \tag{12}$$

where δ_o is defined in the previous chapter, and g is a unique /9
function of the radius.

On examining Equation (9), it is reasonable to define η as follows:

$$\eta = \frac{y}{\delta} = \frac{y}{\delta_0 g} \tag{13}$$

Now it is necessary to determine whether it is possible to find a transformation which will fulfill conditions (9), (10), and (13). Before doing so, let us examine the dimensionless radius

$$q = \frac{r}{r_0} \tag{14}$$

By inserting Equations (7), (8), (9), (10), (13), and (14) into Equation (1), and performing all the necessary differentiations, we obtain the following equation

$$f''' + f'^{2} \frac{Re_0 \frac{\delta_0}{d_0}}{2q^2} \frac{d}{dq}(g q) = 0 \tag{15}$$

where ' denotes differentiation with respect to η, and where

$$Re_0 = \frac{U_{0av} \, d_{H_0}}{\nu} = \frac{4 \, U_{0av} \, \delta_0}{\nu} \tag{16}$$

It is clear that Equation (15) can fulfill the unique dependence of f on η only if the following exists

$$\frac{Re_0 \frac{\delta_0}{d_0}}{2q^2} \frac{d}{dq}(gq) = a \tag{17}$$

where a is a constant. It is easier to deal with the equation if we define

$$\frac{2a}{3\frac{Re_0 \delta_0}{d_0}} = D \tag{18}$$

It is also necessary to define the boundary conditions. One boundary condition is defined in Equation (10). Another boundary condition is a result of the fact that there is no slipping on the plane surface y = 0. A third boundary condition is connected with the shear force on the free surface y = δ, which is proportional to the second derivative of f. In this case, it is possible to assume that this force is zero because of the low viscosity of the air, relative to liquids under normal conditions. It is possible now to formulate the problem by the following equations /10

$$f''' + a f'^2 = 0 \tag{19}$$

with the boundary conditions

$$\eta = 0 \quad f' = 0 \tag{20a}$$

$$\eta = 1 \quad f = 1 \tag{20b}$$

$$f'' = 0 \tag{20c}$$

$$\frac{d}{dq}(gq) = 3Dq^2 \tag{21}$$

with the boundary conditions

$$q = 1 \qquad g = 1 \tag{22}$$

The velocities u and v are defined by the functions f and g, since when we insert (9), (13), and (14) into (7) and (8), we get

$$u = u_{0_{av}} \frac{f'}{g q} \tag{23}$$

$$v = u_{0_{av}} \frac{\delta_0}{r_0} \eta f' \frac{g'}{g q} = u \frac{\delta_0}{r_0} \eta g' \tag{24}$$

3. Solution of Layer Thickness

Equation (21) is immediately integrable:

$$g q = D q^3 + c_1$$

or, after inserting the boundary condition (22)

$$g = D q^2 + \frac{1-D}{q} \tag{25}$$

In Figure 2 we show the function g for the value D = 0.726, which fits formula 15a in appendix A.

In practice, the minimum will depend on the value of D, or a, which has not been fixed yet and which depends on the solution of f. But, in this case, we prefer to choose that value of D because, in the author's opinion, it is very close to the true value of D (see Appendix A), and therefore, the behavior of the

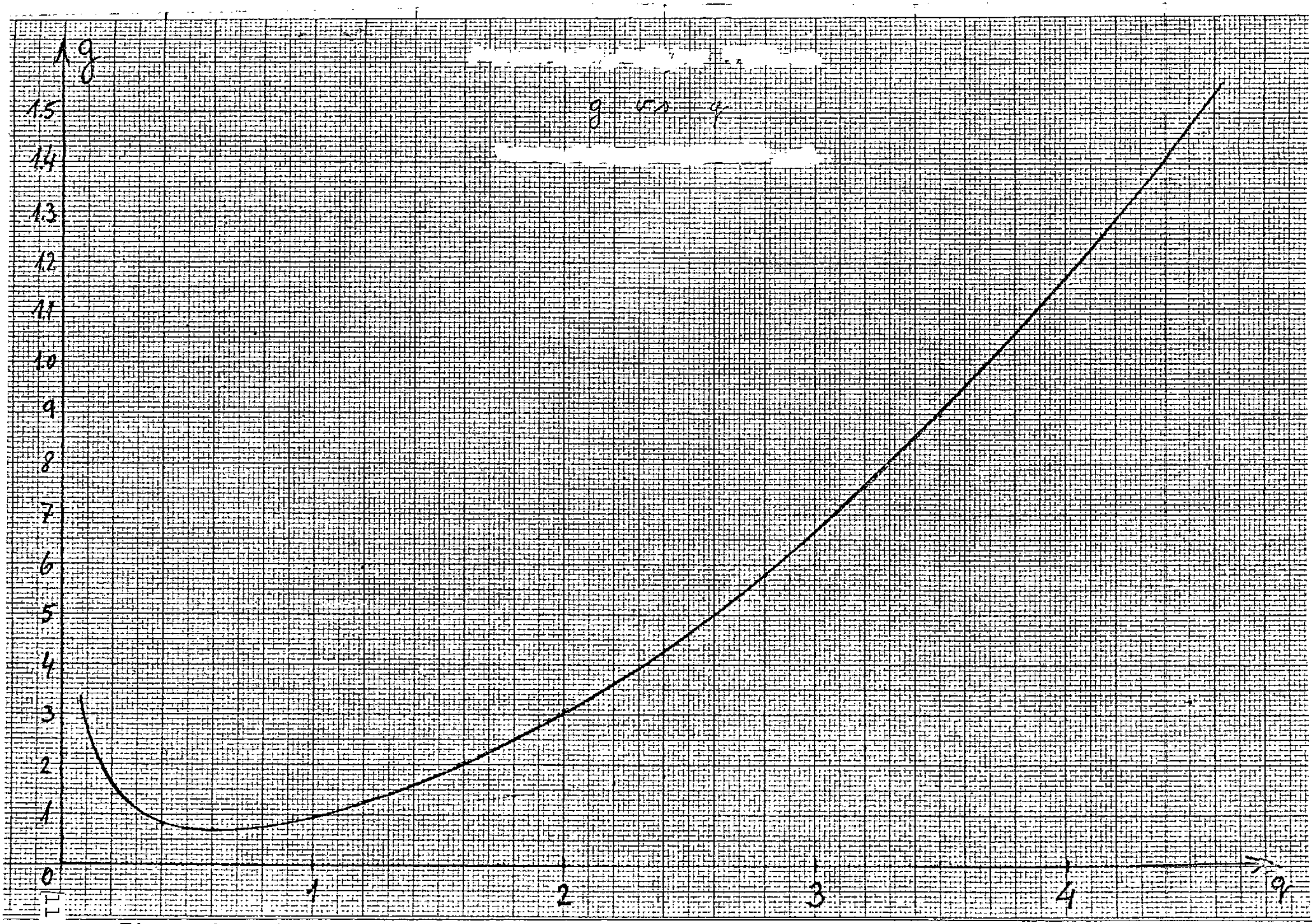

Figure 2. Dimensionless layer thickness.

function g will resemble the curve in Figure 2. /12

For values of D close to the mentioned value, it is found that the reduction of the velocity, because of the shear forces, is so strong that not only is there a radial expansion of the layer, but the layer thickness increases with the distance from the impingement point.

4. Solution of the Stream Function

Equation (19) does not have a closed solution. It was solved by the expansion

$$f = A_0 + A_1 \eta + A_2 \eta^2 + \dots \tag{26}$$

and by inserting (26) into (19) and using the boundary conditions (20a) and (20b), one gets

$$A_0 = A_1 = A_3 = A_4 = \dots = A_{3j} = A_{3j+1} = \dots = 0 \tag{27}$$

$$\left.\begin{aligned} A_5 &= -\frac{a A_2^2}{15} \\ A_8 &= \frac{a^2 A_2^3}{252} \\ &\vdots \\ A_{3j+2} &= -\frac{a}{(3j+2)(3j+1)3j} \sum_{k=0}^{j-1} \left[(3k+2)(3j-3k-1)A_{3k+2}A_{3j-3k-1}\right] \end{aligned}\right\} \tag{28}$$

By inserting (20c) and the connection (11), which is derived from the definition of the stream function, it is possible to fix the numerical value of A_2 and a as follows:

$$
\left.\begin{aligned} A_2 &= 1.1376 \\ a &= 1.788 \end{aligned}\right| \qquad (29)
$$

and the solution is

$$
\left.\begin{aligned} f = {} & 1.13995\,\eta^2 - 1.57\times10^{-1}\,\eta^5 + 1.9339\times10^{-2}\,\eta^8 \\ & - 2.4234\times10^{-3}\,\eta^{11} + 3.0284\times10^{-4}\,\eta^{14} - 3.78568\times10^{-5}\,\eta^{17} \\ & + 4.732\times10^{-6}\,\eta^{20} - 5.915\times10^{-7}\,\eta^{23} + 7.394\times10^{-8}\,\eta^{26} \end{aligned}\right| \qquad (30)
$$

5. Analysis of the Solutions /13

The function f and its derivatives f' and f" are shown in Figure 3. One sees that f is quite close to being linearly dependent on η, and that the boundary condition (28) has a marked effect on the value of f' only close to the limit $\eta = 1$. Choosing a different value for f" (1) will not have a big effect on the values of f' and f for sufficiently small values of η. It is possible to use this analysis for various values of f" (1) and obtain solutions which will converge for decreasing values of η.

We may mention that one possible solution was suggested by Glauert [5], for the case where the viscosity and density of the liquid equals that of its surroundings.

Assumption (28) has other results. It fixes the value of the coefficient appearing in the formula for g (25) when the numerical value of g depends only on the average velocity Uo_{av}, and the layer thickness δ_o within the diameter d_o.

It is worthwhile to mention again that Equations (25) and (30) describe the flow field only when $\delta/d \rightarrow 0$, and when δ/d approaches sufficiently large values (close to the impingement point), the present solution loses its accuracy and is not useful any more.

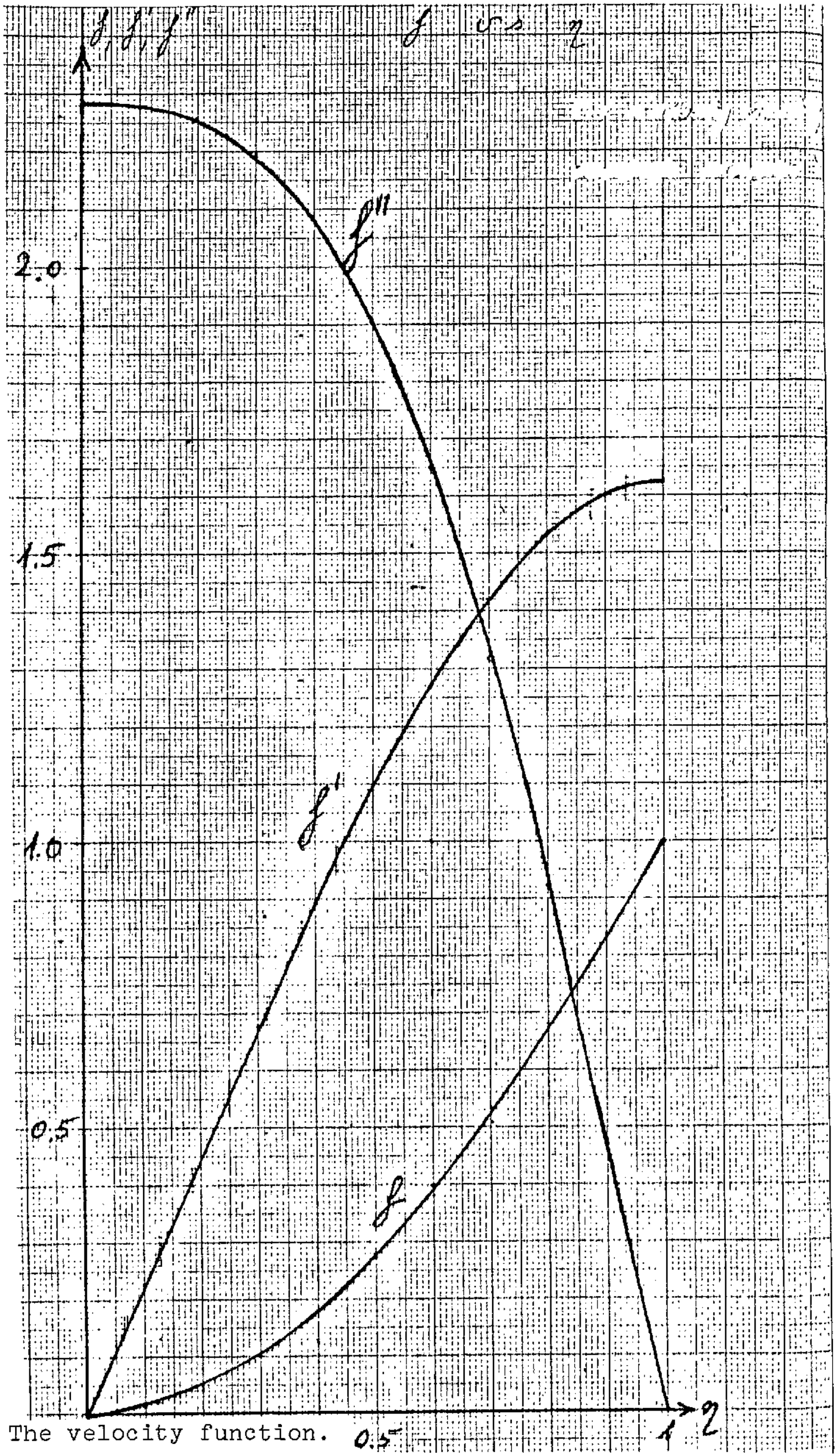

Figure 3. The velocity function.

Inserting the solution (25), in order to solve δ/d, yields

$$\frac{\delta}{d} = \frac{g\delta_0}{q d_0} = \frac{2a}{3Re_0} q + \frac{3Re_0}{2a} \quad \frac{1-D}{D} \quad \frac{1}{q^2} \tag{31}$$

It is clear, therefore, that the existance of the solution depends first of all on Re_0 being sufficiently large. For sufficiently large values of q, the solution is inadequate.

These considerations make it possible to define (depending on the accuracy required) the boundary of zone c as described in Chapter B-1, where this solution exists.

CHAPTER C

ANALYTICAL SOLUTION OF THE HEAT TRANSFER PROBLEM

1. Statement of the Energy Equation

The solution of the energy equation in zone c relies on the above solution of the momentum equation in the same zone, and therefore all the general assumptions described in Chapter B-1 are relevant here.

Using these assumptions, and using order of magnitude analysis, led us in Chapter B-1 to formulate Equations (1) and (2) as residues of the momentum equations for axially symmetric flow. In a similar way, it is possible to develop the following equation from the general energy equation for axial symmetry flow:

$$u \frac{\partial t}{\partial r} + v \frac{\partial t}{\partial y} = \alpha \frac{\partial^2 t}{\partial y^2} \tag{32}$$

With the boundary condition

$$r = r_0 \qquad t = t_0(y) \tag{33a}$$

$$y = 0 \qquad t = \bar{t}\,(r) \tag{33b}$$

$$y = \delta \qquad q_t = 0 \tag{33c}$$

The boundary conditions (33a) and (33b) are general and it is possible to fit them to different cases. The boundary condition (33c) is less general and has the physical meaning that the amount of heat transferred by radiation and free convection from the flowing liquid to its surrounding is negligible relative to the total amount of heat transferred from the solid surface to the liquid. It is clear that this condition exists only at sufficiently low temperatures and when the jet velocity and thermal diffusivity (α) are large.

It is necessary to insert in Equation (32) and boundary condition (33) the values u and v according to (18), (23), (24), and (25), and perform a transformation from a system with coordinates y, r to a system with coordinates η, q, by using Equations (11) and (14). As a result, the energy equation is given by the following expression:

$$\frac{\partial^2 t}{\partial \eta^2} = \frac{1}{2} Re_0 \, Pr \frac{\delta_0}{d_0} f' \frac{g}{q} \frac{\partial t}{\partial q} = \frac{a \, Pr}{30} f' \frac{g}{q} \frac{\partial t}{\partial q} \qquad (34)$$

and the boundary conditions (33a) and (33b) become

$$q = 1 \qquad t = t_0(\eta) \qquad (35a)$$

$$\eta = 0 \qquad t = \bar{t}(q) \qquad (35b)$$

The boundary condition (33c) states that the surface of the liquid is an adiabatic plane. But in the q-η plane, the velocity does not have a component in the η direction, but only in the q direction, and therefore we have /16

$$\frac{\partial t}{\partial \eta} = 0 \qquad \eta = 1 \qquad (35c)$$

while, from heat balance considerations (Figure 4), one gets for nonadiabatic conditions

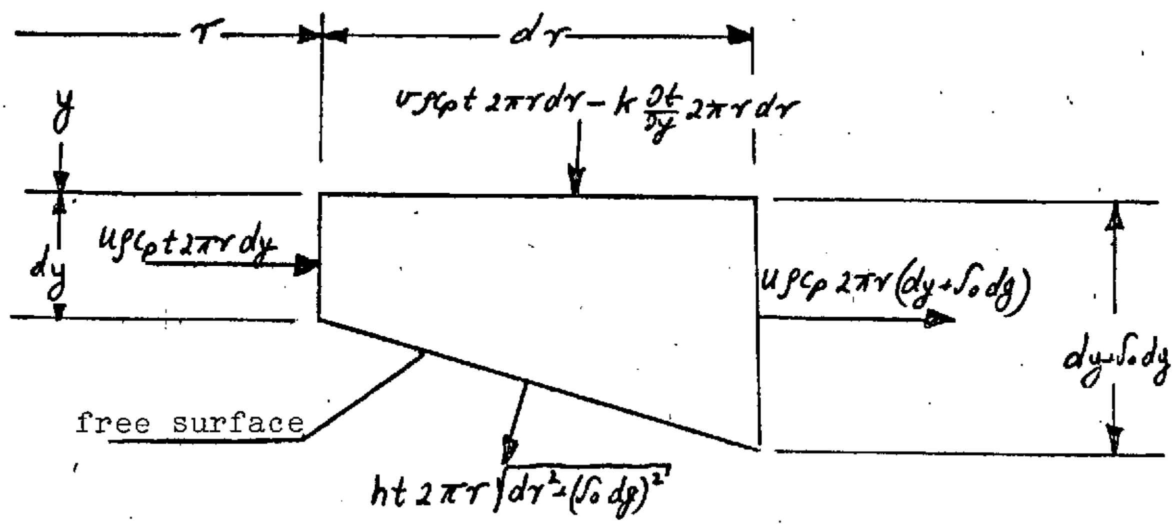

Figure 4. Heat transfer near a free surface.

$$\frac{\partial t}{\partial \eta} = -\frac{h\delta_0}{k}\, g \sqrt{1+\left(\frac{\delta_0}{r_0}\;\frac{dg}{dq}\right)^2} \tag{35c*}$$

It is desirable to insert a dimensionless expression for the temperature

$$\theta = \frac{t-t_i}{t_0-t_i} \tag{36}$$

and because of the homogeneity of Equation (34), we get the final equation with the boundary condition

$$\frac{\partial^2\theta}{\partial\eta^2} = \frac{1}{4}\, Re_0\, Pr\, \frac{\delta_0}{r_0}\, f'\, \frac{g}{q}\cdot\frac{\partial\theta}{\partial q} \tag{37}$$

$$q = 1 \qquad \theta = \theta_0(\eta) \tag{38a}$$

$$\eta = 0 \qquad \theta = \bar{\theta}(q) \tag{38b}$$

$$\eta = 1 \qquad \frac{\partial\theta}{\partial\eta} = 0 \tag{38c}$$

2. The Separation of the Variables and Its Physical Significance /17

In this chapter we shall try to perform a separation of variables in Equation (37), and find the boundary condition (38) for making this separation possible. To do so, it is necessary to assume that θ is a product of a function of η times a function of q

$$\theta = Y(\eta) \cdot Q(q) \tag{39}$$

By inserting (39) into (37) and performing separation of variables we get the equation

$$\frac{Y''}{f'Y} = \frac{1}{4} Re_0 \, Pr \, \frac{\delta_0}{r_0} \, \frac{g}{q} \, \frac{Q'}{Q} = \lambda \tag{40}$$

and the boundary conditions will be

$$q = 1 \qquad \theta_0(\eta) = Y \, Q(1) \tag{41a}$$

$$\eta = 0 \qquad \bar{\theta}(q) = Y(0) \, Q \tag{41b}$$

$$\eta = 1 \qquad Q \frac{\partial Y}{\partial \eta} = 0 \tag{41c}$$

and immediately it is clear that relations (41a) and (41b) are a necessary condition for separation of variables.

Assuming that these conditions exist, the full formulation of the problem is as follows:

$$Y'' = \lambda f' Y \tag{42}$$

$$\eta = 0 \qquad Y = 1 \tag{43a}$$

$$\eta = 1 \qquad Y' = 0 \tag{43b}$$

$$Q' = \frac{\lambda}{\frac{1}{4} Re_0 \, Pr \, \frac{J_0}{\eta_0}} \, \frac{q}{g} \, Q \qquad (44)$$

$$q = 1 \qquad Q = 1$$

3. Derivation of the Surface Temperature

By inserting g from (25) and D from (18) into Equation (44), we get the solution

$$Q = C_1 \, (g q)^{\frac{\lambda}{a Pr}} \qquad (45)$$

and by inserting the boundary condition (44) together with (22), we get

$$\bar{\theta}(q) = Q(q) = (g q)^{\frac{\lambda}{a Pr}} \qquad (46)$$

We find the ratio /18

$$\lambda = a \, Pr \, \frac{\ln \bar{\theta}(q)}{\ln (g q)} \qquad (47)$$

and if λ, which is calculated according to (47), is constant (and does not change with q), it is possible to perform separation of variables by using Equation (25) and the following binomial results from (46):

$$\bar{\theta}(q) = \left(D q^3 + 1 - D\right)^{\frac{\lambda}{a Pr}} = \left(D q^3\right)^{\frac{\lambda}{a Pr}} \left[1 + \frac{\lambda}{a Pr} \, \frac{1-D}{D q^3} + \ldots\right]$$

where

$$D q^3 > 1 - D$$

One sees immediately that, if the value of λ and Pr fulfill the condition

$$\frac{3\lambda}{a Pr} \approx \beta$$

then when q is sufficiently large, $\bar{\theta}$ is approximated by

$$\bar{\theta}(q) = D^{\beta/3} q^{\beta}$$

and so the shape of $\bar{\theta}$ depends on the parameter β.

4. Derivation of the Layer Temperature

The temperature distribution in the flowing layer is described by the function Y which is given by (42) and (43). In the case of a large Prandtl number, it is possible to assume that most of the temperature gradient is close to the wall, and it is possible to retain only the first part of f' and assume

$$f' = 2.28\eta + \ldots\ldots$$

Then we get the equation

$$Y'' = A\eta y \tag{48}$$

with the boundary condition (43), where λ is given by (47), and

$$A = 2.28\lambda \tag{49}$$

This equation can be solved by the expansion

$$Y = \sum_{n=0}^{\infty} \frac{A^n}{L(n)} \left[\frac{a_0}{K(n)} + \frac{a_1}{M(n)} \eta \right] \eta^{3n} \tag{50}$$

where /19

$$\begin{aligned} K(n) &= 2\cdot5\cdot8\ldots\ldots(3n-1) \\ L(n) &= 3\cdot6\cdot9\ldots\ldots 3n \\ M(n) &= 4\cdot7\cdot10\ldots\ldots(3n+1) \\ K(0) &= L(0) = M(0) = 1 \end{aligned} \tag{51}$$

and by inserting the boundary condition (42), one gets

$$a_0 = 1 \tag{52}$$

$$a_1 = -\frac{\sum_{n=0}^{\infty} \frac{A^n}{K(n)\,L(n-1)}}{\sum_{n=0}^{\infty} \frac{A^n}{L(n)\,M(n-1)}} \tag{53}$$

The values $\frac{a_1}{\lambda}$ as function of A are shown in Figure 5. The function Y itself is drawn in Figure 6 for various values of A, and it is clear that if A is sufficiently large, the approximation of a linear distribution of velocities is a good one, since the temperature gradient close to the wall is large.

Explicitly, one obtains the following formula:

$$Y = 1 + a_1\eta + \left(\frac{1}{2\cdot 3} + \frac{a_1}{3\cdot 4}\eta\right)\eta^3 + \left(\frac{1}{2\cdot 3\cdot 5\cdot 6} + \frac{a_1}{3\cdot 4\cdot 6\cdot 7}\eta\right)\eta^6 \tag{54}$$

By inserting (39), (46), and (54), the final result is

$$\theta = \frac{t - t_i}{\bar{t}_0 - t_i} = (g\,q)^{\frac{\lambda}{a P_r}}\left[1 + a_1\eta + \dots\right] \tag{55}$$

5. Derivation of the Heat Flux and the Heat Transfer Coefficients

By using Equations (13) and (55), it is possible to calculate /22 the local heat flux

$$q_t = -k\left.\frac{\partial t}{\partial y}\right|_{y=0} = (-a_1)\frac{k}{\delta_0}(\bar{t}_0 - t_i)\frac{(g\,q)^{\frac{\lambda}{a P_r}}}{g} \tag{56}$$

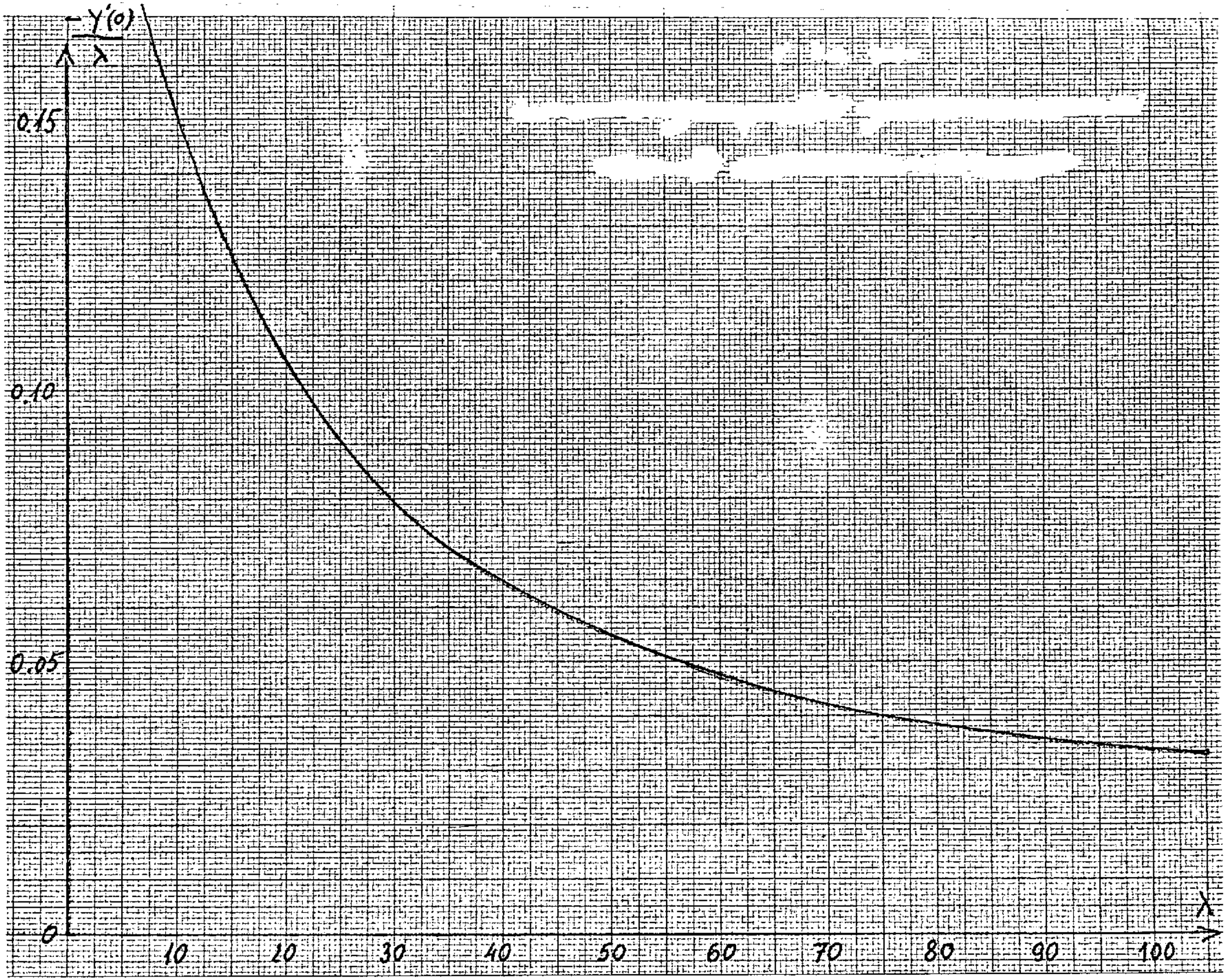

Figure 5. Calculation of temperature gradient near the wall.

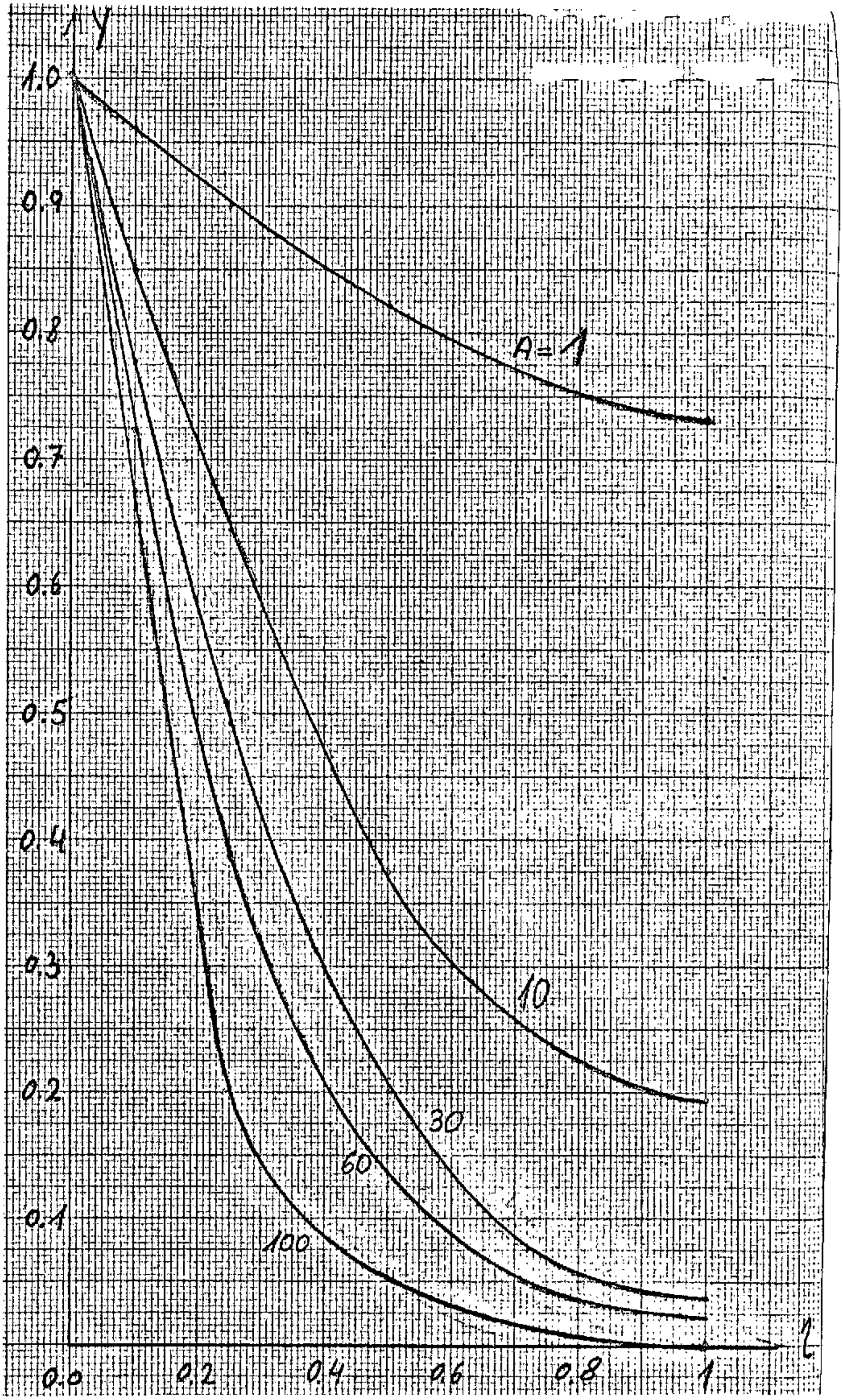

Figure 6. The temperature function.

and from Formula (56), it is possible to calculate the local film coefficient

$$h = \frac{q_t}{\bar{t}_o - t_i} = (-a_1) \frac{k}{r_o} \frac{(g\,q)^{\frac{\lambda}{aP_r}}}{g} \tag{57}$$

and also the local Nusselt number

$$N_u = \frac{h\,r_o}{k} = (-a_1) \frac{(g\,q)^{\frac{\lambda}{aP_r}}}{g} \tag{58}$$

and the mixing temperature

$$t_{mix} = \frac{\int_0^r 2\pi r u t\, dr}{\int_0^r 2\pi r u\, dr} \tag{59}$$

To achieve this goal, one has to calculate the dimensionless expression

$$\theta_{mix} = \frac{\int_0^1 q u \theta\, d\eta}{\int_0^1 q u\, d\eta} = (g\,q)^{\frac{\lambda}{aP_r}} \frac{\int_0^1 f' Y\, d\eta}{\int_0^1 f'\, d\eta}$$

which can be reduced to

$$\theta_{mix} = \left[-Y'(0)\right] \frac{\bar{\theta}}{\lambda} = (-a_1) \frac{\bar{\theta}}{\lambda} \tag{60}$$

By means of (60), one can calculate the mixing temperature

$$t_{mix} = t_i + \theta_{mix} \left(\bar{t}_o - t_i\right) \tag{61}$$

Because of the practical importance of Equation (61), it is useful to insert in it Equations (46) and (60) and obtain

$$t_{mix} - t_i = \left(\frac{-a_1}{\lambda}\right) \left(\bar{t} - t_i\right) \tag{62}$$

where $(-a_1)$ is defined by (51) and λ by (47).

It is important to remember that the solution exists only under the conditions described in Chapter C-2 and formulated by the boundary condition (41). /23

Although one is not always sure that these conditions exist, usually the influence of the initial condition is limited to a certain distance, and therefore, for a sufficiently large q, the solution may even describe a situation where the boundary conditions do not fulfill (41).

CHAPTER D
THE EXPERIMENTAL APPARATUS

1. The Purpose of the Experiments /24

At the same time that the theoretical work of the research was under way, an experiment was performed to check qualitatively and quantitatively the nature of the physical phenomena in order to justify the theoretical assumptions and their regions of validity.

Various phenomena were checked in this part of the research, which relate to the problems of flow and heat transfer. Also, the practical physical phenomena were analyzed, and a quantitative comparison was performed between the calculated and experimental results.

2. Description of the Apparatus

The experimental work was performed in a special apparatus built for that purpose. The apparatus is shown in detail in Figures 7 and 8 and in photos 1 and 2.

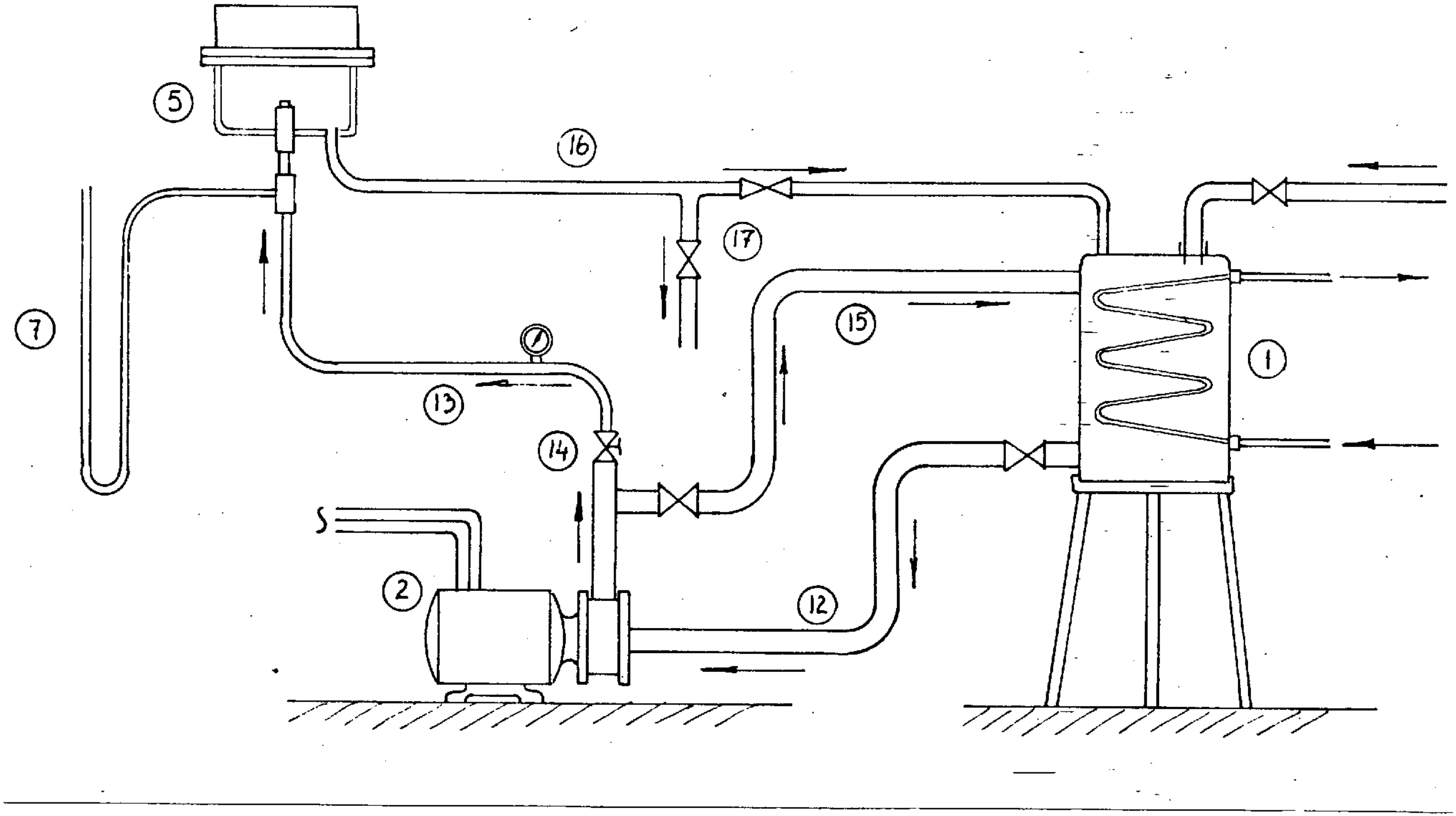

Figure 7. Scheme of the experimental apparatus.

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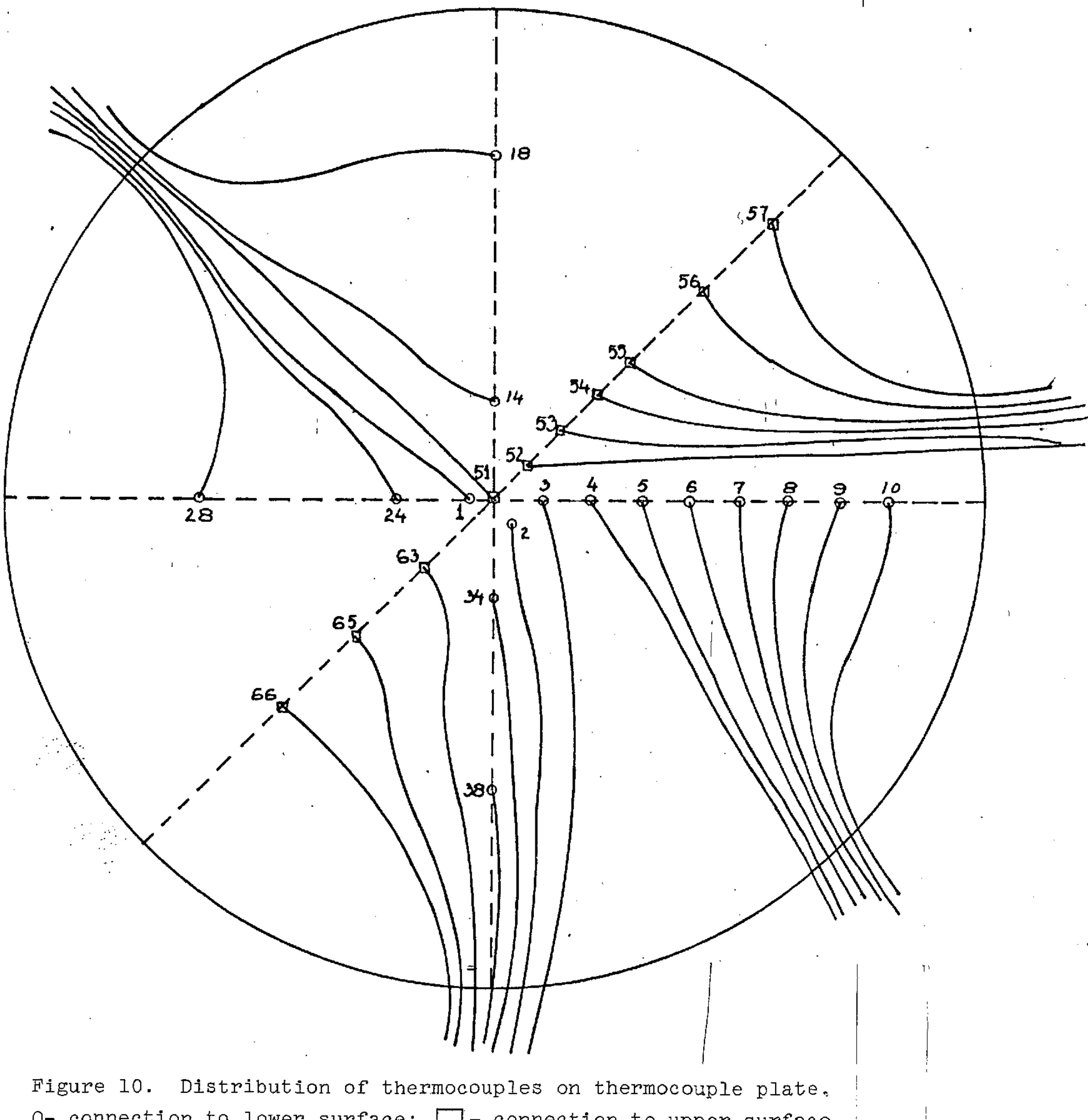

Figure 10. Distribution of thermocouples on thermocouple plate.
O- connection to lower surface; □- connection to upper surface.

3. Description of the Experiments

The experiments performed included the impingement of a viscous liquid jet on heated and unheated surfaces, and a qualitative and quantitative check of the thermal field and the flow. A solution of glycerin and water was chosen, since it was easily possible to create low and high concentrations and since this solution has a relatively high viscosity.

The experiments were divided into two series as follows:

a) checking the shape of the flow. In the first series of measurements, the general shape of the flow was checked. To do so, the thermocouple plate 5 was interchanged with a transparent plate. By splashing the jet from below on the transparent plate, it was possible to observe the general shape of the flow. By attaching the surplus line (14) to the container 1 high above the liquid surface, small bubbles of air were introduced into the liquid, and it was possible to watch them accompanying the flow through the transparent plate. In this way, it was possible to follow the flow lines in the layer, and get a qualitative idea about the velocity in the layer.

b) checking the temperature field: in this series of measurements, all the different parts of the apparatus were assembled, and the effect of the jet impingement from below on the temperature field of the thermocouple plate which was heated from above, was checked.

For that purpose, the quantity of liquid, flowing through the jet, was measured by a graduated cylinder and stop watch (using the set of valves 17), the specific gravity of the liquid was measured, and the temperature at the jet orifice and where it leaves the apparatus was measured. The temperature was also measured by means of the potentiometer 11 from both sides of the thermocouple plate, and the separation diameter of the jet from the board was noted.

CHAPTER E
RESULTS OF THE EXPERIMENTS-FLOW /30

1. Qualitative Description of the Flow

The first purpose of the experiments was to create a laminar flow, and to check the characteristics of this flow relative to the flow described in Chapter B-1. The experiments described in paragraph a of Chapter D-3 served this purpose. Photographs were taken of the flow through a transparent plate (Photos 4, 5, 6, 7, and 8), where one can identify clearly the flow lines. The various zones which are described in Chapter B-1 are clearly seen in these pictures.

The central zone (fits zones a and b).

Zone c - where a radial flow exists on top of the solid surface, and where the radial component of the velocity u is much greater than the perpendicular component of the velocity v. In this zone, there is not a marked effect of the surface tension and gravitational forces.

In zone d the velocity is already very small, since most of the momentum is transferred into heat and therefore gravitational and surface tension forces are the main acting forces. Equilibrium between these two forces determines the diameter of separation of the liquid from the plane surface.

It is possible to identify two forms of separation: a) the separation of the liquid as a continuous film (Photo 4) having a conic envelope, with a wide base resting on the separation diameter on top of the plane surface, and a narrow base on top of the holder of the orifice; and b) the separation of the liquid at some points on the separation diameter (Photos 5, 6, 7, 8), where in each such point a vertical jet is created pointing downward.

According to the observations, the number of separation points is fixed so that the distance between them is always constant (balance between surface tension and gravity).

Photo 4. $Re_i = 1612$; $d_i = 0.6$ cm.*

2. Determination of the Separation Diameter /31

As a result of the measurements performed, it was found that the thickness of the flowing layer at the separation point is fixed and does not depend on the flow parameters. It is important to note that the measurements were performed at almost constant values of density and surface tension (of the flowing liquid), and therefore, this behavior is anticipated, since the separation occurs only after the "decay" of the velocity. This phenomena enables one to calculate the diameter of separation.

Photo 5. $Re_i = 1127$; $d_i = 0.6$ cm.*

Calculation of the dependence of the separation diameter on /32 the surface tension and specific gravity is outside the scope of this research, and will not be dealt with.

In the following, we shall calculate the layer thickness at the separation point by means of Equations (16), (18), (25):

$$\delta_s = g_s \delta_0 = \delta_0\left[D q_s^2 + \frac{1-D}{q_s}\right] \approx \frac{2.38}{Re_i}\left(\frac{r_s}{r_i}\right)^2 r_i \qquad (63)$$

* Translator's note: Photos not supplied.

Photo 6. $Re_i = 930$; $d_i = 0.6$ cm.* /33

TABLE OF RESULTS

Experiment No.	r_i cm	Re_i	q_s	δ_s cm
13	0.1095	1448	4.49	0.184
14	0.1095	798	4.31	0.207
16	0.051	546	5.15	0.156
17	0.1095	1633	4.94	0.214
21	0.1095	1062	4.41	0.197
24	0.271	462	2.28	0.172
25	0.271	685	2.66	0.205
26	0.271	1132	2.82	0.195

The resulting average value of the separation diameter is 0.191 cm.

Photo 7. $Re_i = 563$; $d_i = 0.1$ cm.* /34

Photo 8. $Re_i = 375$; $d_i = 0.1$ cm.*

CHAPTER F
RESULTS OF EXPERIMENTS — HEAT TRANSFER

The temperature difference $t_{mix} - t_i$ is calculated by the formulae

$$\lambda = 1.788\, P_r \frac{\ln \bar{\theta}_s}{\ln q_s \delta_s} \qquad (47)$$

* Translator's note: Photos not supplied.

$$A = 2.28\lambda \tag{49}$$

according to Figure 4,

$$-\frac{a_1}{\lambda} = f(A) \tag{53}$$

$$t_{mix} - t_i = \left(\frac{-a_1}{\lambda}\right)\left(\bar{t} - t_i\right) \tag{62}$$

The results are compiled in the accompanying table. The large difference between the measured and calculated values of $t_{mix} - t_i$ are explained by the fact that the difference was not directly measured but rather calculated as a difference between two measurements. Since the voltage produced by the thermocouples, which is a measure of the value $t_{mix} - t_i$, is between 0.01 and 0.04 millivolts, the accuracy could not be improved.

It is worthwhile noting that the average of the error is

$$1 - \Sigma \frac{(t_{mix} - t_i)_{mean}}{(t_{mix} - t_i)_{3/3N}} = 5.9\%$$

Another interesting result is the distribution of the isotherms in the thermocouple plate (Figure 11), for experiment number 26. These lines were calculated by relaxing the equation of heat transfer to an axially symmetric body

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) + \frac{\partial^2 t}{\partial y^2} = 0 \tag{64}$$

where the boundary conditions for this equation are inserted from the experimental measurements.

It is obvious that the heat transfer takes place via forced convection to the jet, while the free convection in a diameter greater than the separation diameter does not play any role in this phenomenon.

CHAPTER G
CONCLUSION

1. Summary

After checking the flow conditions in a laminar jet impinging /38 perpendicularly on a flat surface, it became clear that it is possible to divide the jet into zones having different flow behavior. It was found out that the dominant zone which contributes the most to heat transfer is the zone where the main component of the velocity is parallel to the flat surface. In this zone one may neglect some parts from the momentum equation, and get a solution to the energy equation (when some of the velocity values and the thickness of the flowing layer in a certain diameter in this zone are known). The solution indicates that the flowing layer thickness reaches its minimum, and then grows parabolically as function of the dimensionless diameter q. The velocity profile is an almost linear function of the dimensionless thickness η. The thickness of the flowing layer at the separation point was calculated using the developed formulas, and it was checked for all the cases that its value is a fixed number, i.e., 0.191 cm, as one may anticipate in light of the fact that the density and surface tension are almost constant in these experiments. By inserting the solution of the velocity into the energy equation, we calculated the temperature and heat transfer coefficients, as a function of the flowing liquid characteristics, the jet conditions and the surface temperature. The average difference between the theoretical calculations and the experimental results is about 6%, and therefore it seems reasonable to suggest that this method

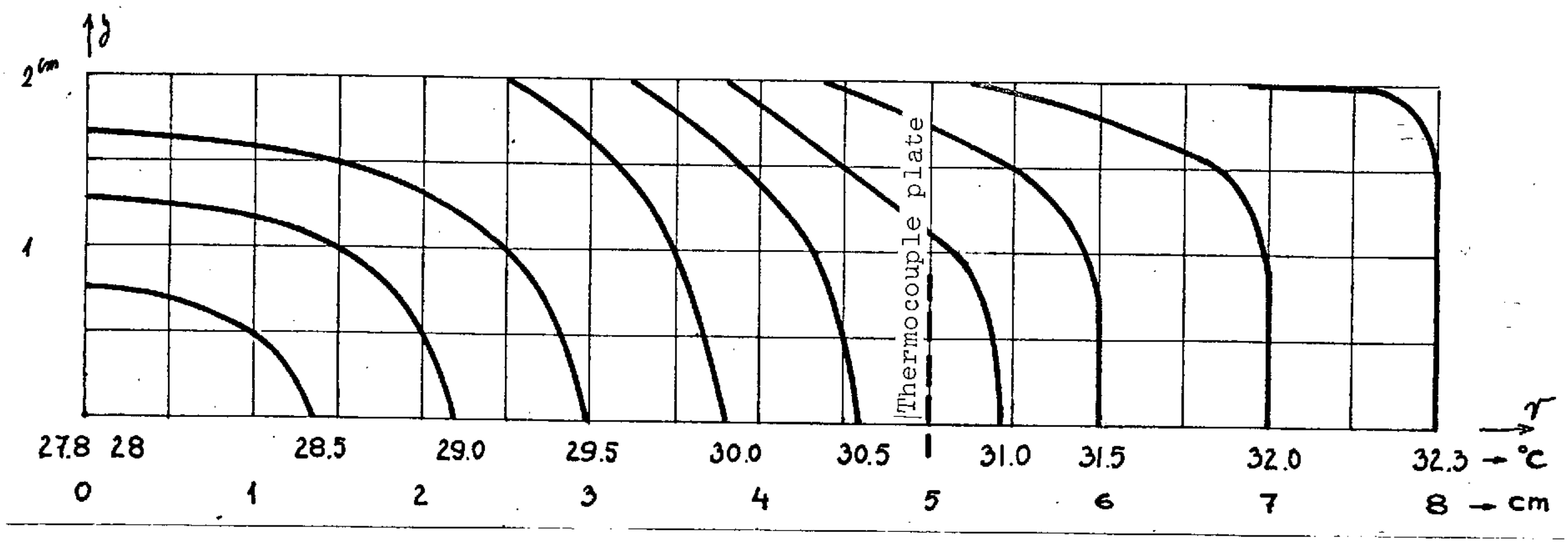

Figure 10. Isotherms in the thermocouple plate.

TABLE OF RESULTS

HEAT TRANSFER

$t_{mix}-t_i$ Measured	$t_{mix}-t_i$ Calculated	$-\frac{a_1}{\lambda}$	A	λ	t_o-t_i	$\bar{\theta}_s$	g_s	q_s	P_r	Re_i	r_i	No.
°C	°C				°C						cm	
0.298	0.332	0.0475	60.2	26.4	3.9	1.795	14.7	4.49	106	1448	0.1095	13
1.540	0.505	0.0500	56.1	24.6	6.1	1.657	13.5	4.31	111	798	0.1095	14
0.869	1.057	0.059	45.1	19.8	11.3	1.584	19.4	5.15	111	546	0.051	16
0.372	0.380	0.052	53.8	23.6	3.5	2.085	17.9	4.94	81	1633	0.1095	17
1.118	0.628	0.061	43.2	18.9	5.9	1.746	14.2	4.41	79	1062	0.1095	21
0.645	0.813	0.072	34.0	14.9	8.9	1.269	3.9	2.28	77	462	0.271	24
0.421	0.468	0.060	44.2	19.4	5.4	1.426	5.2	2.66	81	685	0.271	25
0.298	0.275	0.052	53.2	23.3	3.3	1.607	5.9	2.82	73	1132	0.271	26

of calculation developed in this work is appropriate for calculating heat transfer between an impinging jet and a flat surface.

2. Recommendations on Further Research

Not all the subjects mentioned in this research were explored and it is necessary to continue as follows:

Experimental subjects:

measurements should be taken of the flowing layer thickness;

accurate measurements should be taken of the difference between the temperature of the incoming flow and at the mixing cup.

Theoretical subjects:

the solution of the stream in the impact zone;

the solution for the case of liquids having characteristics which are functions of temperature;

it is also desirable to continue the research of temporarily unstable jets, of jet impingement on surfaces which are not perpendicular to the jet, and on simultaneous impingement of several jets.

APPENDIX A

CALCULATION OF THE INITIAL CONDITIONS AT DIAMETER d_o

The theoretical solution to the flow field described in this work is valid only for distances sufficiently far from the impingement point, when the thickness of the flowing layer is greater than this distance. In order to make numerical calculations, it was necessary to estimate the average velocity and the thickness of the layer at diameter d_o. In order to do so, we assumed the existence of a velocity field as described in Figure A-1, where the velocity field at a diameter smaller than d_o is the potential velocity field with a viscous boundary layer. At diameters greater than d_o, it changes into a viscous velocity field as described in Chapter B of this paper. The mentioned potential field is special in the sense that on the free surface of the liquid (which constitutes a streamline), the pressure is constant. An exact solution to such a velocity field has not been found yet, and therefore, we tried to use the solution to a stagnant flow which does not give a constant pressure along an external streamline. The mentioned solution was found by Frössling [17], and quoted by Schlichting [16]. This solution deals with a flow which impinges on a plane perpendicular surface, which disperses on it radially, and it describes accurately only that portion of the flow which is close to the jet center. In the intermediate zone, which is between the zone where a solution of the above problem exists, and the zone where the solution is given by this paper, one has an unknown flow field, Anyway, from Figure 2A one may see a marked resemblance between the form of ϕ' which describes the radial velocity of the suggested solution, and between the form of f' which describes the radial velocity of the solution described in this paper. Therefore, we

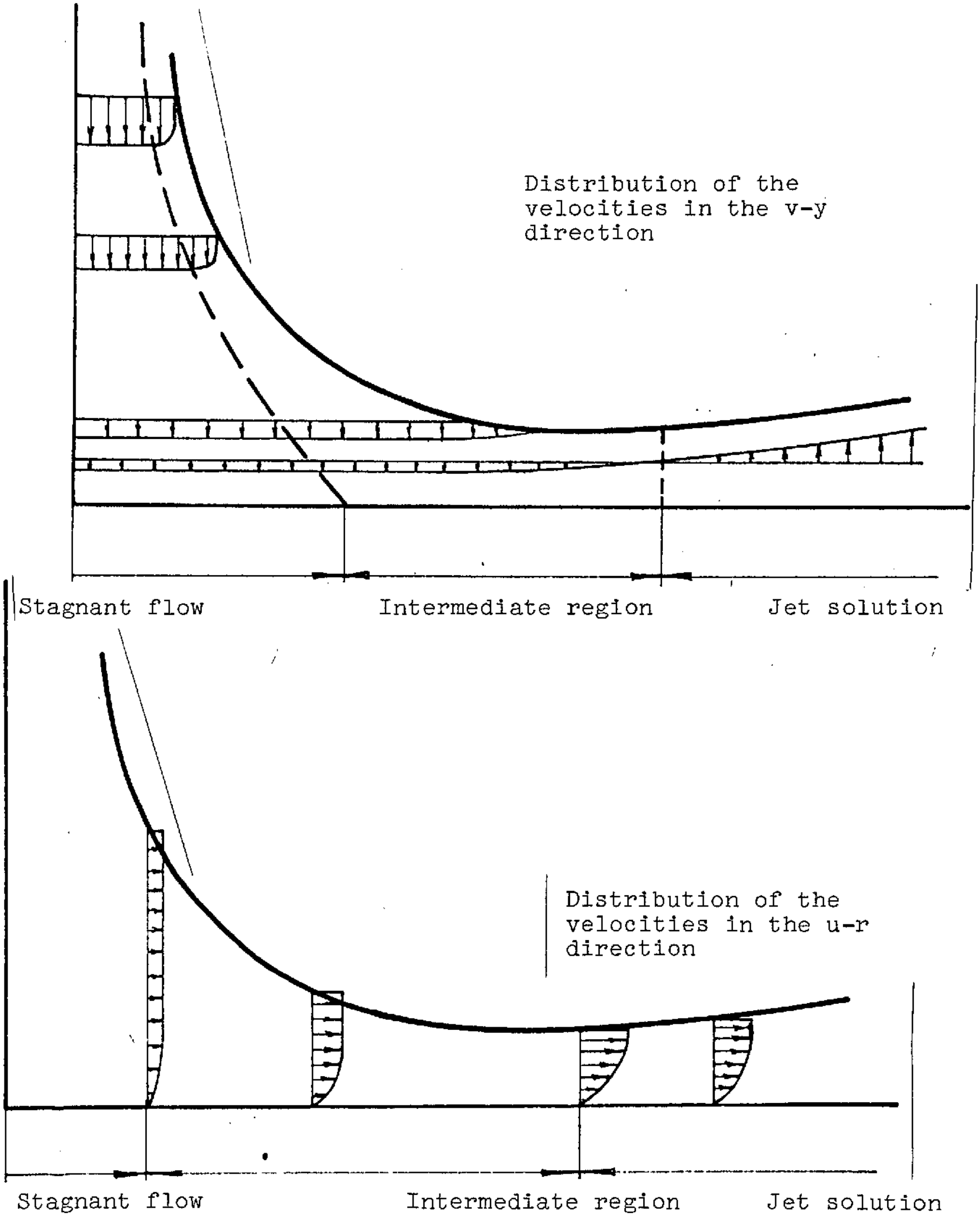

Figure A-1. Qualitative description of the velocity field in the impinging jet.

assumed the existance of a radial velocity in the intermediate zone.

The solution for the stagnant flow is

$$u = a r \, \phi'(\zeta) \tag{1a}$$

where

$$\zeta = y \sqrt{\frac{a}{\nu}} \tag{2a}$$

But in a flow as described here, /42

$$u = \frac{U_{0av} \, f'(\eta)}{g \, r} \tag{23}$$

where

$$\eta = \frac{y}{\delta_0 \, g} \tag{11}$$

and from Figure 2-A, one can see that by chosing

$$\zeta \sqrt{2} = 2\eta \tag{3a}$$

one obtains

$$\frac{\phi'\left(\frac{\zeta}{\sqrt{2}}\right)}{0.9422} \approx \frac{f'(\eta)}{1.626} \tag{4a}$$

and the following two relations are established:

$$y = \zeta \sqrt{\frac{\nu}{a}} = \eta \, \delta_0 \tag{5a}$$

$$U_0 = U_{0av} \, f'(\eta) = a \, r_0 \frac{1}{\sqrt{2}} \, \phi'\left(\frac{\zeta}{\sqrt{2}}\right) \tag{6a}$$

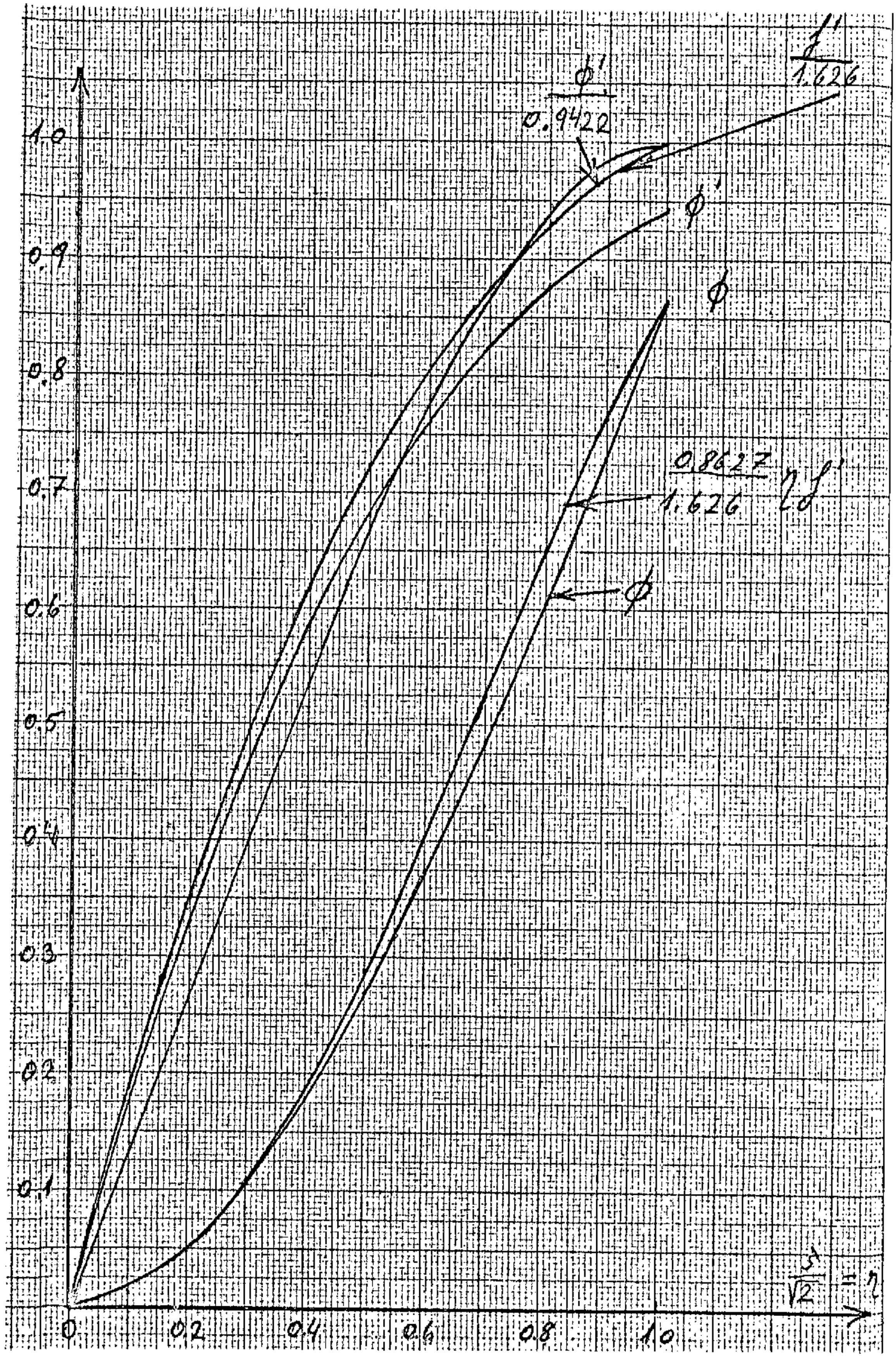

Figure 2-A. Comparison of velocity profiles in stagnation and jet flow.

which can be reduced by using (93-a) and (4-a) to

$$\delta_0 = \sqrt{\frac{2\nu}{a}} \tag{7a}$$

$$U_{0_{av}} = \frac{0.9422}{1.626\sqrt{2}} \, a \, r_0 \tag{8a}$$

Since the velocity of the surface of the liquid inside the diameter d_o has a potential, one has

$$U_0(1) = 1.626 \, U_{0_{av}} \approx V_i = \frac{Q}{\pi r_i^2} \tag{9a}$$

and the continuity equation is satisfied

$$Q = \pi d_0 \delta_0 U_{0_{av}} \tag{10a}$$

A simultaneous solution of Equations (7a), (8a), (9a), and (10a) yields the following results

$$U_{0_{av}} = 0.785 \frac{Q}{d_i^2} \tag{11a}$$ /43

$$\frac{r_0}{r_i} = 0.63 \, Re_i^{1/3} \tag{12a}$$

$$\frac{\delta_0}{r_i} = \frac{1.296}{Re_i^{1/3}} \tag{13a}$$

$$a = \frac{2.38}{Re_i^{1/3}} \frac{V}{r_i} \tag{14a}$$

Equations (12a) and (13a) are described in Figure 3-A. Inserting Equations (11a), (12a), and (13a) in Equation (25) yields the accurate value of g

$$g = 0.726\, q^2 + \frac{0.274}{q} \tag{15a}$$

It is important to note that g does not depend on the Reynolds number but only on q, but q itself depends on the Reynolds number, according to the formula

$$q = \frac{1}{0.63\, Re_i^{1/3}} \quad \frac{\gamma}{\gamma_i} \tag{16a}$$

The function g is described in Figure 2.

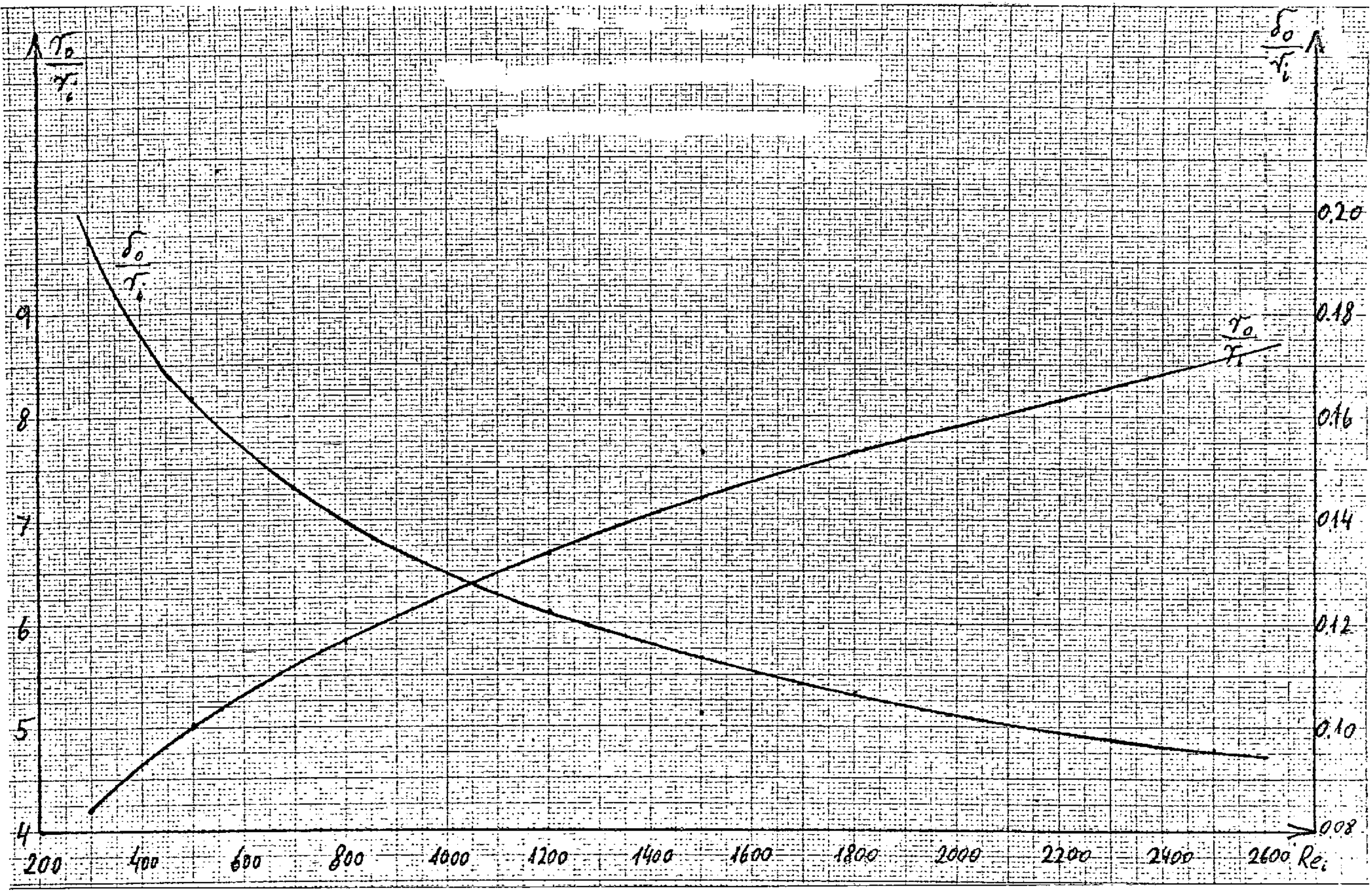

Figure 3-A. Zero point diameter and thickness.

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Translated for National Aeronautical and Space Administration under contract No. NASw-2483 by SCITRAN, P. O. Box 5456, Santa Barbara, California, 93103.

STANDARD TITLE PAGE

1. Report No. NASA TT F 15,373	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle HEAT TRANSFER BETWEEN LAMINAR LIQUID JET AND A FLAT SURFACE		5. Report Date March 6, 1974
		6. Performing Organization Code
7. Author(s) M. Wolfshtein		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address SCITRAN Box 5456 Santa Barbara, CA 93108		11. Contract or Grant No. NASw-2483
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Israel Institute of Technology, M.Sc. Thesis, Technion, Haifa, 1964, 45 pages.

16. Abstract

A description is given of the results of research on heat transfer and flow phenomena in a liquid jet impinging on a flat perpendicular surface. The various zones of flow inside the impinging jet are examined, and the general character of the flow in each of them is established.

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified - Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	~~47~~ 53	5.50